Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

$$=\frac{a^3}{2}(\tfrac{1}{3}+\tfrac{1}{5})-\left[\frac{a^3\sin^6\phi}{12}\log\frac{1+\cos\phi}{\sin\phi}\right]_0^{\frac{1}{2}\pi}-\frac{a^3}{12}\int_0^{\frac{1}{2}\pi}\sin^6\phi\left(\frac{\cos\phi}{\sin\phi}+\frac{\sin\phi}{1+\cos\phi}\right)dx$$

$$=\tfrac{4}{15}a^3-\tfrac{1}{12}a^3\int_0^{\frac{1}{2}\pi}[\cos\phi\sin^5\phi+\sin^5\phi(1-\cos\phi)]d\phi=\tfrac{4}{15}a^3-\frac{a^3}{12}.\frac{4.2}{5.3}=\tfrac{2}{9}a^3.$$

Similar solutions were received from *GEORGE LILLEY*, *LON C. WALKER*, *G. B. M. ZERR*, and *J. SCHEFFER*.

MECHANICS.

127. Proposed by F. P. MATZ, Sc. D., Ph. D., Professor of Mathematics and Astronomy in Defiance College, Defiance, Ohio.

Develop the Fourier Series to represent the temperature of a circular wire of uniform cross-section, in which the temperatures of the four quadrants are in order $t, 2t, 3t, 4t$.

Solution by G. B. M. ZERR, A.M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

From Fourier's *Analytical Theory of Heat*, page 217, we get for the complete solution of the linear and varied movement of heat in a ring after a time T the following:

$$v=e^{-hT}\left[\frac{1}{2\pi}\int_0^{2\pi}f(x)dx+\frac{1}{\pi}\sum_{n=1}^{n=\infty}e^{-n^2kT}\sin nx\int_0^{\frac{1}{2}\pi}\sin nxf(x)dx\right.$$

$$\left.+\frac{1}{\pi}\sum_{n=1}^{n=\infty}e^{-n^2kT}\cos nx\int_0^{\frac{1}{2}\pi}\cos nxf(x)dx\right],$$

$$\text{where } f(x)=\frac{1}{2\pi}\int_0^{2\pi}f(x)dx+\frac{1}{\pi}\sum_{n=1}^{n=\infty}\sin nx\int_0^{2\pi}\sin nxf(x)dx$$

$$+\frac{1}{\pi}\sum_{n=1}^{n=\infty}\cos nx\int_0^{2\pi}\cos nxf(x)dx.$$

$$\int_0^{2\pi}f(x)dx=t\int_0^{\frac{1}{2}\pi}dx+2t\int_{\frac{1}{2}\pi}^{\pi}dx+3t\int_{\pi}^{\frac{3}{2}\pi}dx+4t\int_{\frac{3}{2}\pi}^{2\pi}dx=5\pi t.$$

$$\int_0^{2\pi}\sin nxf(x)dx=t\int_0^{\frac{1}{2}\pi}\sin nxdx+2t\int_{\frac{1}{2}\pi}^{\pi}\sin nxdx+3t\int_{\pi}^{\frac{3}{2}\pi}\sin nxdx+4t\int_{\frac{3}{2}\pi}^{2\pi}\sin nxdx.$$

$$=\frac{t}{n}(\cos\tfrac{1}{2}\pi n+\cos\pi n+\cos\tfrac{3}{2}\pi n)-\frac{3t}{n}=-\frac{4t}{n},\text{ except when } n=4m.$$

$$\int_0^{2\pi}\cos nxfxdx=t\int_0^{\frac{1}{2}\pi}\cos nxdx+2t\int_{\frac{1}{2}\pi}^{\pi}\cos nxdx+3t\int_0^{\frac{3}{2}\pi}\cos nxdx+4t\int_{\frac{3}{2}\pi}^{2\pi}\cos nxdx$$

$$=-\frac{t}{n}(\sin\tfrac{1}{2}\pi n+\sin\tfrac{3}{2}\pi n)=0.$$

$$\therefore f(x)=\tfrac{5}{2}t-(t/\pi)(4\sin x+2\sin 2x+\tfrac{4}{3}\sin 3x+\tfrac{4}{5}\sin 5x+\tfrac{4}{6}\sin 6x+\tfrac{4}{7}\sin 7x+\tfrac{4}{9}\sin 9x+\ldots)$$

$$\therefore\ V=e^{-hT}[\tfrac{5}{2}t-(t/\pi)(4\sin xe^{-kT}+2\sin 2xe^{-2^2kT}+\tfrac{4}{3}\sin 3xe^{-3^2kT}+\tfrac{4}{5}\sin 5xe^{-5^2kT}+\ldots)]$$

AVERAGE AND PROBABILITY.

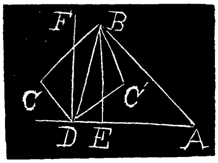
---

108. **Proposed by A. H. HOLMES, Brunswick, Me.**

Required the average area of the quadrilateral whose sides are a, b, c, and d.

I. **Solution by G. B. M. ZERR, A. M., Ph.D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.**

Let $ABCD$ be the quadrilateral, $AB=a$, $AD=b$, $BC=c$, $CD=d$, $a>b>c>d$, $BE=v$, $DE=u$,.



When the quadrilateral is convex, the average area is, (the average area of ABD) + (the average area of BCD).

When the quadrilateral is concave, the average area is, (the average area of ABD) − (average area of $BC'D$).

Since BCD is equal to $BC'D$, the average area required is the average area of $ABD=\triangle$. Area $ABD=\frac{1}{2}ab\sin A$.

Now $DB=\surd(u^2+v^2)$.

$\therefore \cos A=(a^2+b^2-u^2-v^2)/2ab$.

$\therefore$ Area $ABD=\frac{1}{4}\surd[4a^2b^2-(a^2+b^2-u^2-v^2)^2]$.

But $v^2=a^2-(b-u)^2$.

$\therefore$ Area $ABD=\frac{1}{2}b\surd[a^2-(b-u)^2]=C$.

The limits of u are 0 and $[(c+d)^2+b^2-a^2]/2b=u'$.

$$\therefore \triangle=\frac{\int_0^{u'}C\,du}{\int_0^{u'}du}=\frac{b}{2u'}\int_0^{u}\surd[a^2-(b-u)^2]du.$$

$$\therefore \triangle=\frac{b^2}{(c+d)^2+b^2-a^2}\Big[\frac{b}{2}\surd(a^2-b^2)+\frac{a^2}{2}\sin^{-1}\frac{b}{a}-\frac{a^2+b^2-(c+d)^2}{8b^2}$$

$$\surd\{4a^2b^2-[a^2+b^2-(c+d)^2]^2\}-\frac{a^2}{2}\sin^{-1}\Big(\frac{a^2+b^2-(c+d)^2}{2ab}\Big)\Big].$$